Comment on "Force Balance at the Transition from Selective Withdrawal to Viscous Entrainment"

J. Eggers† and S. Courrech du Pont‡

†*School of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, United Kingdom*
‡ *Laboratoire Matière et Systèmes Complexes, UMR CNRS 7057, Université Paris Diderot*
10, rue Alice Domon et Léonie Duquet, 75205 Paris cedex 13, France

In a recent letter [2], Blanchette and Zhang (BZ) proposed a theory for the critical flow rate at the selective withdrawal transition. Their theory is based on the assumption of failure of the interface on a large scale, presumed insensitive to the viscosity μ_0 of the entrained fluid. We show that BZ's theory is untenable, as it is inconsistent with the hydrodynamic description they use, and also disagrees with experiments done in a fluid-air system in the same geometry [3], in which no failure was observed, and entrainment only occurs when the highly deformed tip of the interface enters the orifice, see Fig. 1, top inset.

We performed our own numerical simulations of the hydrodynamic equations, reported in Fig. 1. We show the critical flow rate at a viscosity ratio $\mu_0/\mu = 1$ (solid line), using an infinite domain (for details, see [5]), as appropriate for the very large tank used in the two-fluid experiment [1]. Our results are in perfect agreement with earlier numerical calculations [4] for large S/ℓ_c (+). In the lower inset we show the (linearly) unstable mode at failure for $S/\ell_c = 1.3$, leading to entrainment. Contrary to BZ's claim, the unstable mode is highly localized near the central peak, and much more so as μ_0/μ is decreased to $1/10$. Thus viscous hydrodynamics, used as the basis for BZ's argument, predicts an entrainment mechanism inconsistent with their central assumption of delocalized failure, nor can the viscosity ratio be neglected, as the most unstable mode depends strongly on it.

We now discuss our experiments in a fluid-air system [3], in which *no* instability was observed, and entrainment occurs only when the hump tip enters the tube. If the interior viscosity μ_0 may be neglected, the interface shape is well described by Taylor's theory [5]. Postulating that the deformation occurs over a distance $r \propto \ell_c$, one finds for the flow rate as the hump height is maximum ($H = S$) that $Q_c = a\gamma S^3/\mu\ell_c$. This law is shown as the dotted line in Fig. 1, with $a = 2.2$, which agrees very well with our experiment for the fluid-air system (*). One can see in Fig. 2 of BZ that the transition occurs in a parameter range where the hump height H is very sensitive to small variations of Q, so that the S-value at the transition is close to the marginal case $H = S$ (realized in the fluid-air experiments). This explains the good alignment of the data from [6] with S^3 scaling.
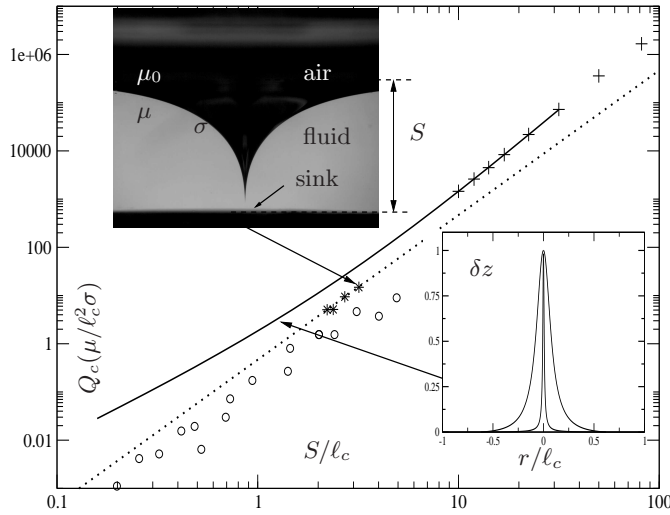
Notice that the solid line lies considerably *above* the critical flow rates of the fluid-fluid experiment (o), and even above the fluid-air data (*). We believe the latter to be due to finite size effects, which deform the interface away from the tip (see Fig. 17 of [5]). As for the fluid-fluid data, taken in a very large tank, it remains unclear which experimental feature is missing from the hydrodynamic description, as discussed in detail in [5]. We believe the agreement between theory and experiment reported by BZ to be an artefact of the unphysical boundary condition imposed at the edge of a very small domain, chosen arbitrarily to be of size ℓ_c. This amounts to an adjustable parameter, and thus an arbitrary shift in the x-direction. In conclusion, the arguments of BZ do not explain the experimental data.

FIG. 1: Critical flow rate versus distance between sink and the unperturbed interface S, in units of the capillary length ℓ_c. The line is our numerical simulation for $\mu_0/\mu = 1$, o: data from [1] as shown in [2], ∗: data from [3]; Q multiplied by 3 [3], +: simulation from [4]. Dotted line: $Q_c = 2.2\gamma S^3/\mu\ell_c$. Top inset: image of fluid-air interface as the tip enters the tube, [3]. Bottom inset: failure mode δz for $S/\ell_c = 1.3$ and $\mu_0/\mu = 1$ as well as $\mu_0/\mu = 0.1$ (narrow peak).

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